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Detection of the special gravity signals in sub-tidal band by using wavelet technique

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Abstract Based on the 5-year length of tidal gravity observations recorded with a superconducting gravimeter at Wuhan International Tidal Gravity Reference Station, the special gravity signals associated with the possible Earth's solid inner core translational oscillations in sub-tidal bands are detected and studied by using for the first time a wavelet transformation technique. The analysis is conducted on gravity residuals after removing the synthetic tidal gravity signals and air pressure perturbation from original observations, demonstrating that there exist gravity oscillation signals at 4—6 h bands with amplitude of nGal level. However, it is found that the frequency and amplitude of such kind of oscillation signals change with time, and the analysis shows that these oscillation signals are provoked probably by some non-continuous source with very low amplitude.

Keywords: gravity signals in sub-tidal band, wavelet time-frequency spectrum, Earth's solid inner core, Slichter modes.

With the rapid increase of the requirements to geodynamics of the Earth's deep interior in global basic sciences and space techniques, the theoretical study and experimental detection of the geodynamic phenomena in Earth's liquid core (ELC) and Earth's solid inner core (ESIC) have become now more and more important. The dynamics of the Earth's interior now has become nowadays one of the most important and basic problems among 25 international frontier ones in Earth's science. In recent years, the great progress has been made in study of the dynamics problem of the ELC, however, only some valuable attempts are done in study of the translational oscillations of the ESIC. Nowadays, there are no convincing theoretical models and complete detection results. The past researches have shown that the gravity measurement is unique effective method in study of the geodynamical phenomena in Earth's deep interior except for the seismological technique. The translational oscillations of the ESIC, including three parts, equatorial prograde, axial and equatorial retrograde translations, and they are Earth's fundamental free modes usually called Slichter triplets or Slichter modes[1,2]. To study and detect Slichter modes, it is hopeful for us to recognize the characteristics of the detailed structures and density distributions at boundary between Earth's liquid core and solid inner core, by determining accurately their oscillation parameters of the ESIC[3, 4].

The theoretical studies have shown that for a non-rotating, symmetrical and layered Earth's model, the predominant components (also called eigenvectors) of the translational oscillations of the ESIC are described usually with three degree 1 spheroidal displacement vectors. However other kinds of spheroidal and toroidal displacements (with same order but different degrees) are coupled together due to the action of Coriolis force of the Earth's rotation and Earth's ellipticity form. By using a generalized spherical harmonic expansion, the eigenvectors can be expressed theoretically by sum of degree 1 spheroidal vector σ_1^m and degree 2 toroidal displacement vector τ_2^m ($m = -1$, 0 and 1). The numerical eigenperiods were given as 4.916 h, 4.441 h and 4.055 h respectively for equatorial prograde, axial and equatorial retrograde components of the ESIC when adopting a DG597 Earth model[4]. The results obtained during the 1960 Chile large earthquake proved that the stratification in fluid outer core and elastic property of the ESIC, and the gravity signals induced by equatorial retrograde translational mode is at level of 2 nGal[2]. In order to evaluate accurately the truncation errors and to sweep aside the uncertainty of convergence in generalized spherical harmonic expansion, under the suppose of "sub-seismic approximation", and by using a finite element technique in variation approach, Smylie obtained translational parameters of the ESIC, based on the CORE11, the predicted eigenperiods are given as 3.581 h, 3.766 h and 4.015 h and for model 1066A, they are given as 2.603 h, 2.702 h and 2.824 h, differing from those of Smith[3, 4].

The new important techniques and wide prospects in search for the translational oscillations of the ESIC emerge thanks to the successful construction of new superconducting gravimeter (SG) with high precision in GWR company in USA and the starting of Global Geodynamics Projects (GGP). Compared to any spring gravimeters, the SG has many excellent characteristics, including extreme wide linear measuring range, extreme low noise level, extreme high stability with resolution in order of nGal level and sensitivity[5]. Some publications have proved that by using SG observations with high accuracy and high sampling rates from globally distributing stations, especially after eliminating instrumental drifts and other systematic errors, the detection of the geodynamical phenomena can be realized, especially to some super-weak gravity signals arising from dynamic behavior in Earth's deep interior[6—8]. The SG can provide us also with high quality data in long-period seismic and sub-seismic bands, and specially those signals with period longer than 54 min can be used to detect the existence of the Earth's spherical oscillation modes (purely elastic ones)[9—12].

In recent years, only some valuable attempts are made in the experimental detection by using ground-based instruments in study of the translational oscillations of the ESIC. The earliest gravity experimental detection was carried out by Smylie *et al.* in 1993 using a product spectra analysis of 4 SG series of tidal gravity recorded at stations in Europe[13]. By using a product spectrum and a cross spectrum techniques, the translational oscillations of the ESIC were also checked experimentally by Hinderer *et al.* in 1995[14]. The multi-station experimental attempt was made by stacking the SG data from 5 stations all over the world by Courtier *et al.* in 2000[15]. Based on the more SG observations in GGP network and the fast Fourier transform (FFT) technique, the SG data are stacked simultaneously in search of gravity signals associated with possible Slichter modes by Rosat and Sun in 2003[16—18]. The studies of Crossley *et al*. have indicated that the magnitudes of gravity signal of the core modes might be at nGal level which falls inside the SG precision range[19].

The above attempts are based on the conventional discrete Fourier transform (DFT), however, this technique can provide us only with the average spectral estimation of gravity signals in a given time span. It can not provide us simultaneously with characteristics in temporal change of the spectral signal. The recent studies have shown the efficiency in using the wavelet time-frequency spectrum (WTFS) technique, in various Earth science research domains.

The main purpose of this study is to use the WTFS and other techniques to analyse the SG observations with length of 5 years (from January 1, 1998 to December 31, 2002), to detect gravity signals in sub-tidal bands associated with possible translational oscillations of the ESIC and to study the properties of these oscillation signals. It is a valuable attempt for developing further theoretical study, for giving valuable attempts in practical detection and also for providing valuable reference in study of Earth's deep interior and its physical properties.

1 Tidal gravity observations

The SG numbered as T004 was importedand started to record tidal gravity in 1985, which was installed in the laboratory of the Institute of Geodesy and Geophysics (IGG), Chinese Academy of Sciences (CAS). It was sent back to GWR company in USA in 1996 for upgrading and reconstruction, because of ageing of the control electronic parts and lines after more than 10 year's continuous running, and it was renumbered as C032. The instrument was installed successfully in 1997 in Jiufeng Geodynamic Laboratory, and the coordinates are given as 30.52°N, 114.49°E at 80 m elevation[20]. In accordance with instrument upgrading and the GGP regulations, for the daily observation, the new data acquisition system developed by German group (via Jentzsch, University of Jena) was installed.

The gravity card used for data acquisition is equipped inside instrument, an on-board anti-aliasing low-pass filter that conforms the GGP requirement is designed. The filter attenuates signals at frequencies greater than 0.5 Hz by 100 dB, and this configuration is intended for a sampling rate of 1 sample per second. Every 10 s interval samplings of tidal gravity and pressure are recorded after passing the anti-aliasing low-pass filter and sent to a 22-bit digital voltmeter. The good equipment for stabilizing room temperature was installed, in order to keep the yearly change of room temperature at less than ±1°[8, 20].

The SG is a kind of relative gravity instrument, the direct output of the instrument is the change of voltage, and it is necessary for us to complete a calibration to instrument output in order to convert effectively digital output in volt into gravity unit. By using the parallel measurements of the FG5-112 absolute gravimeter

(AG) and SG, the two campaigns (from January 29 to February 1, 1999 and from August 13 to 16, 2000) were carried out for the calibration purpose of SG.

During the data processing, the correction of AG laser light speed, valid instrument adjustment heights were considered, the calibration factor of SG was determined by using a least square polynomial linear fit between SG outputs and absolute gravity measurements[21]. The correction of the phase transfer function determined in laboratory of GWR instrument company is employed during the tidal gravity data analysis.

Before data analysis, the detailed processing to SG original observations has been carried out. The available raw samplings of gravity and pressure to be 10 s are checked carefully by using a Tsoft package developed by Royal Observatory of Belgium and recommended by International Center for Earth Tides (ICET)[22].

The Tsoft allows us to process the time series easily in a fully interactive and graphical way, taking advantage of extended graphics capabilities. Under precondition of not destroying as possible the gravity signals, some possible disturbing signals as spikes, steps and other offsets up to several μGals of unknown origin occurring frequently in original observation are removed carefully. Short-term gaps due to power failures, earthquakes and liquid helium refilling are filled using spline interpolation based on a synthetic tidal gravity model.

By using a least square filtering technique and applying a window function, the 0.5 h rate samplings of tidal gravity and pressure signals are obtained. The gravity residual series is obtained by subtracting the synthetic tidal gravity signal from the 0.5 h sampling series. The pressure-gravity coefficient is determined using a regression technique between change in pressure and gravity residuals. The pressure-induced gravity signal is then removed from original observations. The gravity signal due to the polar motion is eliminated from tidal gravity residuals based on the coordinates predication given by the IERS[23]. The temporal variation of the tidal gravity (Fig. 1(a)), station pressure (Fig. 1(b)) and residual series (Fig. 1(c)) in 0.5 h samplings are given in Fig. 1. The residual time series (Fig. 1(c)) will be used to identify the weak signals in sub-tidal band associated possibly with translational oscillations of the ESIC.

2 Methods of data analysis

The methods used to analyse special gravity signals in sub-tidal band are described briefly as follows:

(1) The technique of the multi-stage filter (MSF)

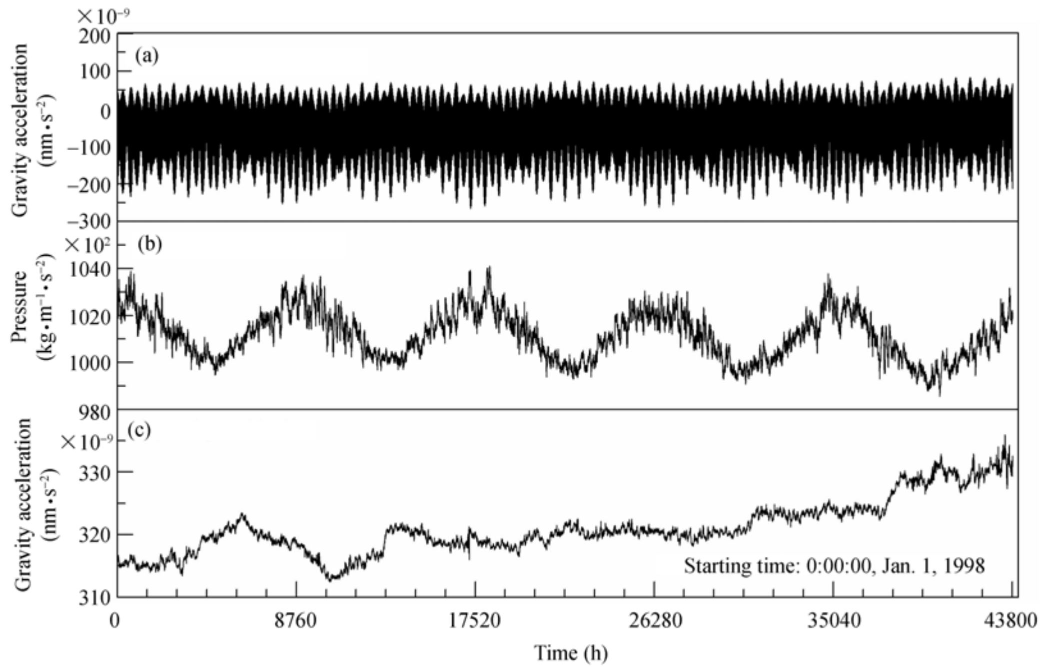

Fig. 1. Temporal variation of tidal gravity (a), pressure change (b) and gravity residuals (c) at Wuhan Station. (January 1, 1998 to December 31, 2002)

based on a multi-combination of Vondrak filters is applied to the first step, the MSF can offer us very high resolutions for certain truncated frequencies[24,25]. In general, the existence of stronger spectral power in low frequency signals down to 10 h period may limit the detection of weak gravity signals in sub-tidal band. Therefore, the MSF is used to eliminate firstly the signals in low frequency band. The gravity signals in sub-tidal band are then identified from the remaining residual series. The theoretical formula of the frequency response function R of the MSF is given as

$$R = c(1 - A(f,e)^L)^M \tag{1}$$

where c is a real constant, taken as 1 in general; L and M are positive integers, determined by the band-width of truncated frequencies; and $A(f, e)$ is the frequency response shown in Vondrak's smoothing method with the following form as:[26]

$$A(f,e) = (1 + e^{-1}(2\pi f)^6)^{-1} \tag{2}$$

where f and e are respectively the corresponding frequency component and smoothing factor. When applying the MSF to gravity remaining residual series, the leap-step time series analysis (LSTSA) model is used to limit the edge effects of the output signals[27].

(2) The WTFS technique is mainly employed to search for gravity signals in range of several hours associated possibly with the translational oscillations of the ESIC. In general, for a given time series expressed as $f(t)$, its wavelet transform can be defined as[28]

$$W_\psi(f)(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-b}{a}\right) dt, \tag{3}$$

where $\psi(t)$ is the basic wavelet; a is the dilation/compression scale factor used to determine the characteristic frequency; and b is the sliding factor in time domain. The normalized Morlet wavelet[29] is applied in our study due to its excellent properties. The real part of the wavelet transform is taken to characterize the spectrum analysis of a real series, and also provide the property of the phase and amplitude for a given signal. Differing from the conventional DFT technique, one of the obvious advantages of the WTFS is to describe the spectrum characteristic of the time series $f(t)$ in time–frequency domains (or a–b space). Therefore, the WTFS will be used to detect the stability and changing procedure of some special gravity signals in sub-tidal bands.

(3) The least squares adjustment of the Householder Transform will be adopted to estimate quantitatively the periods and amplitudes of the gravity signals in sub-tidal wave band detected when using the WTFS technique[30].

(4) By using multi-taper power spectrum for corresponding estimations, the existence of the gravity signals in sub-tidal bands is further determined conveniently[31, 32]. This technique uses multiple windows prior to the Fourier transformation to reduce significantly the spectral leakage in order to provide more reliable spectral estimations for determining special gravity oscillation signals.

3 Spectral estimation of the gravity residuals

The studies have shown that there exist plentiful signals in gravity residuals for wave bands from hours to years. However, what we are interested in are the signals located in sub-tidal band associated with possible translational oscillations of the ESIC. By using the DFT technique, the characteristics of the residual signals in sub-tidal band are checked carefully. It is found that there is effective large energy left in residuals around S_2 and S_1 waves, i.e. 12 h and 24 h with amplitude around nGal level. After applying for pressure correction using a gravity-pressure admittance, there exists still energy at period of 8 h with amplitude at 5 nGal level, but nothing is found at S_4 wave period. The analysis shows that the obvious spectral peaks around S_4 wave were found before pressure correction, but disappeared after applying pressure correction, showing that at 6 h frequency band, the pressure correction is valid. Therefore, no steady signal source corresponding to pressure origin is visible in our interested frequency band.

The estimated WTFS results of tidal gravity residual series (based on Fig. 1(c)) are depicted in Fig. 2. The ordinate is expressed for the spectral period of the signal in hours ranging from 0.1 to 30 days. The signal energy at long periods of monthly, fortnight, diurnal and semi-diurnal wave bands is found obviously. Both magnitudes and periods changing with time and frequency are shown clearly thanks to the excellent WTFS property. The corresponding spectral parameters for these significant signals are identified and estimated by using a method of trial and errors in process of least squares adjustments. The identified periods of the oscillating signals are 27.55 d, 13.65 d, 1.0 d and 0.5 d and their amplitudes are given as 0.161±0.008, 0.130±0.008, 0.123±0.008 and 0.036±0.008 μGal respectively.

The analysis shows that the monthly and fortnight signals relate to the adoption of tidal gravity model in which they are not yet well modeled. Other signals in diurnal and semi-diurnal bands may relate to possible station background noises, such as underground water changes, shallow sea tides, imperfect correction of atmospheric pressure and other unknown reasons. Of course, by analysing independently the same gravity residual series based on the conventional FFT technique, the coincident results in sub-tidal band are obtained.

However, it should be noticed that it is quite difficult for us to identify directly the gravity spectral signals due to the influence of stronger spectral power energy in low frequency band. The analysis shows that the strong oscillating signals on time scales from semi-diurnal to monthly may have been drawn to sub-tidal wave band on the time scales of several hours. Therefore, before the identification of some weak gravity signals in sub-tidal band, it is necessary for us to eliminate as possible the lower frequency signals from residual series. By using the MSF technique (the fre-

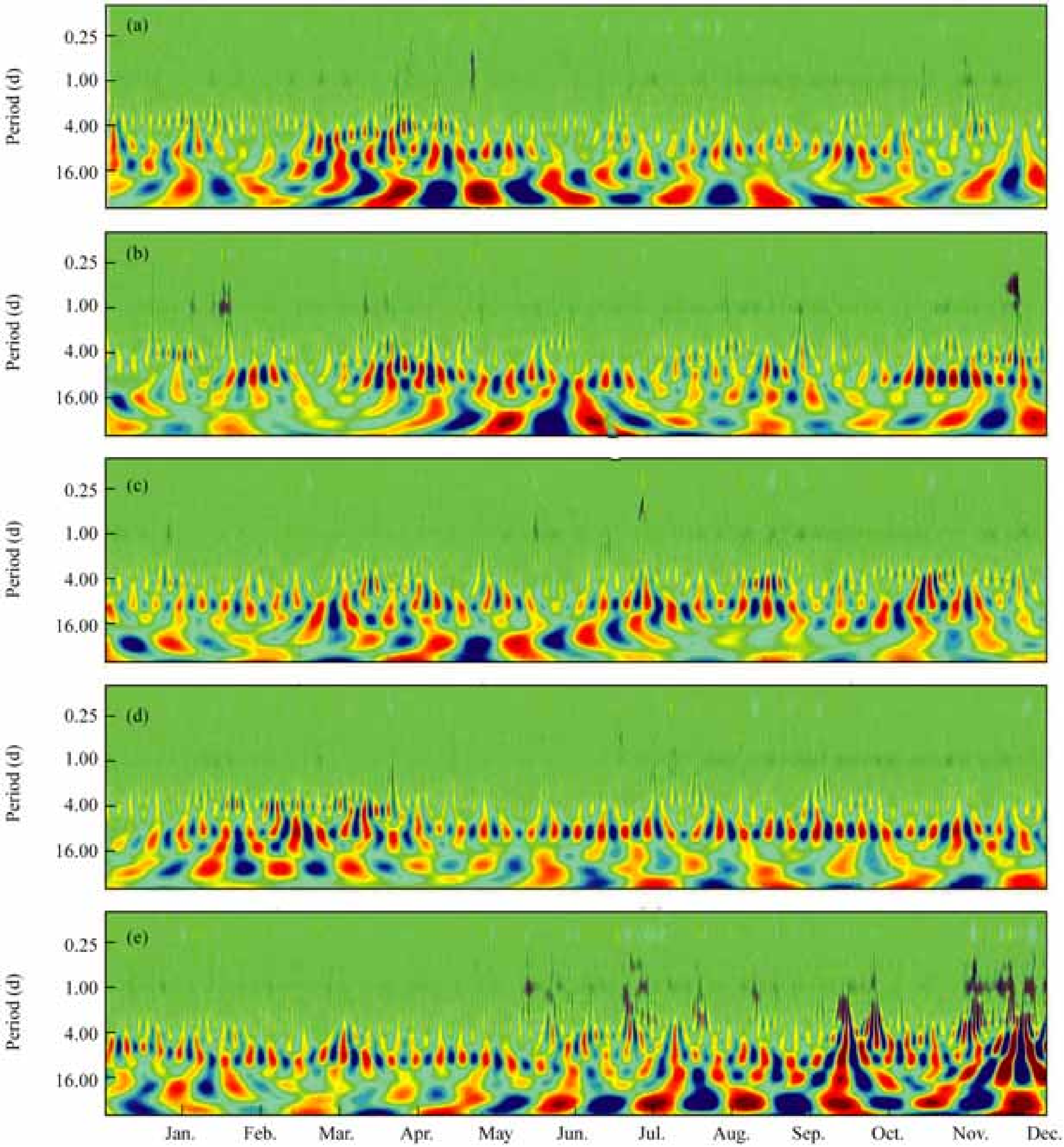

Fig. 2. Property of the wavelet time-frequency spectra of gravity residual series in 1998(a), 1999(b), 2000(c), 2001(d) and 2002(e).

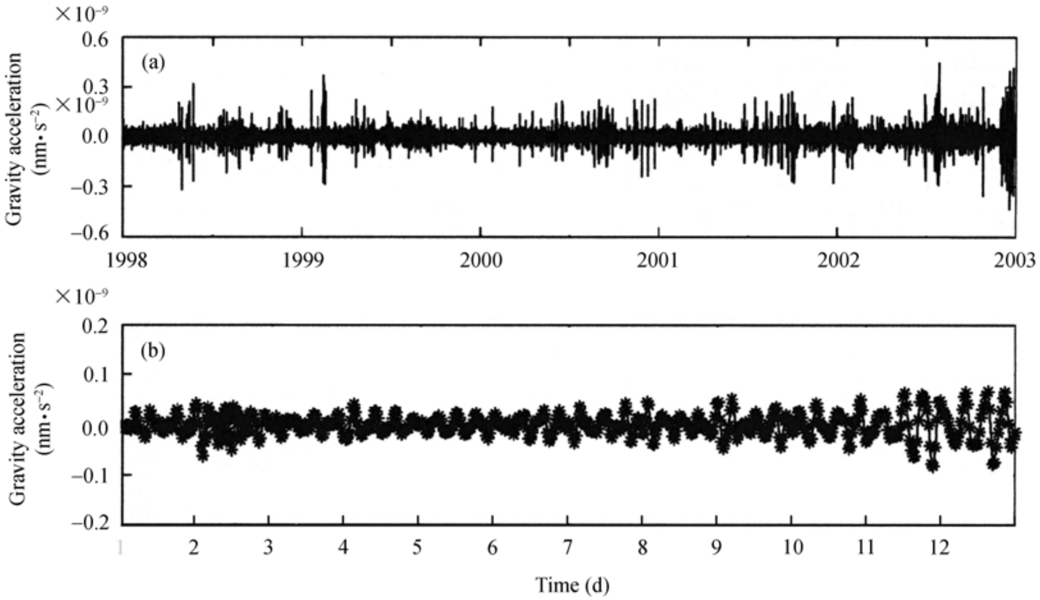

Fig. 3. Time series of high frequency oscillating signals obtained with MSF technique during January 1, 1998 to December 31, 2002 (a) and January 1 to 12, 2001(b).

quency response function for high-pass filter is drawn in Fig. 4), the gravity signal series with a truncated period of 7 h are then obtained, and the filtered gravity residual series are given in Fig. 3(a). In order to observe in more detail those determined oscillating signals, Fig. 3(b) shows the pattern over a time period from January 1 to 12, 2001.

It is found from Fig. 3, particularly from Fig. 3(b), that the oscillation process of gravity signals on the time scales of several hours indeed exists, although their periods and amplitudes seem unstable and vary with time. The analysis shows that the instantaneous amplitudes are weak, which is about one order of magnitude less than those shown in diurnal, fortnight and monthly bands.

The WTFS results of the SG gravity residuals in sub-tidal wave band are depicted in Fig. 5. Each of sub-figures represents the yearly spectral estimates from 1998 to 2002, except that the last one shows the spectral results over a time span from January 1 to 12, 2001 in more detail. It is found in Fig. 5 that the obvious gravity oscillating signals changing with time in the period of 4—6 h exist, although the spectral amplitudes of these gravity signals are unstable.

The spectral analysis of the SG gravity residuals in sub-tidal wave band is also carried out by means of multi-taper spectral estimation technique in order to confirm the results obtained by using the WTFS method. The corresponding results are drawn in Fig. 6.

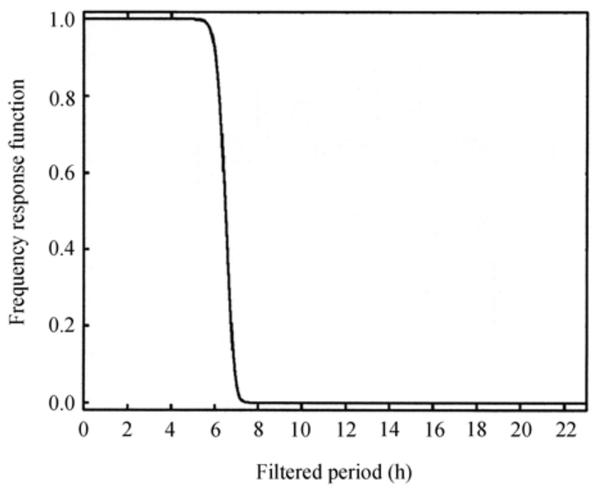

Fig. 4. Frequency response function of the MSF with truncation period of 7 h.

It can be seen easily that the obvious spectral peaks in a wide-band period of 2—6 h are detected, and approach to those we found when using the WTFS technique, with similar characteristics. Unfortunately, the multi-taper spectral estimation technique can’t provide us simultaneously with the property in signal change with time and frequency.

On the other hand, by comparison, the results obtained when using the WTFS technique in Fig. 5 show us not only the signal property varing with time and frequency, but also the higher resolution comparing to the results in Fig. 6 obtained when using multi-taper

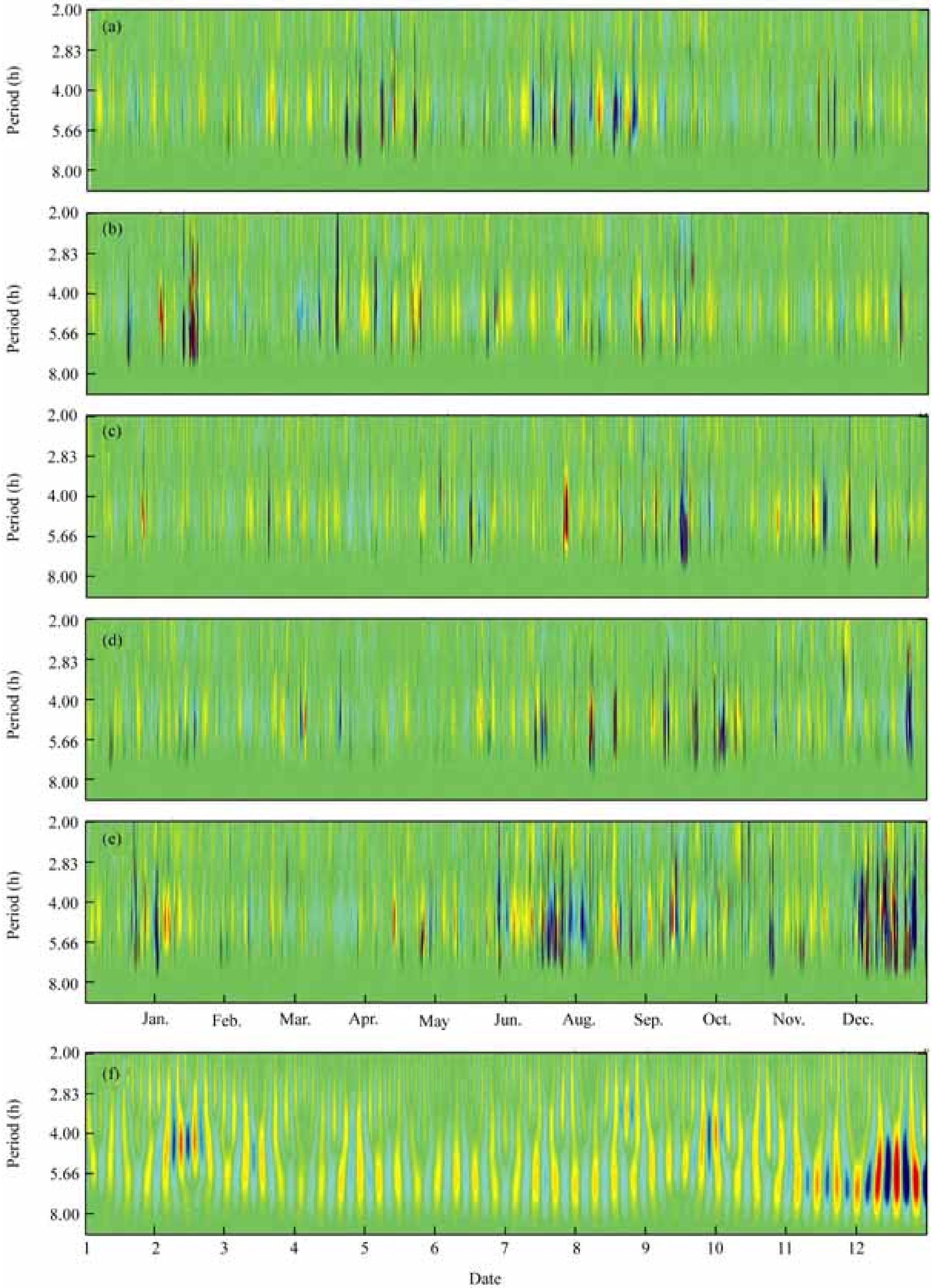

Fig. 5. Property of the wavelet time-frequency spectra of gravity residual series in sub-tidal band in1998 (a), 1999 (b), 2000 (c), 2001(d), 2002 (e) and on January 1－12, 2001(f).

spectral estimation technique. Furthermore, there are no low frequency components in Fig. 6, proving the MSF filter to be very effective in the elimination of low frequency signals.

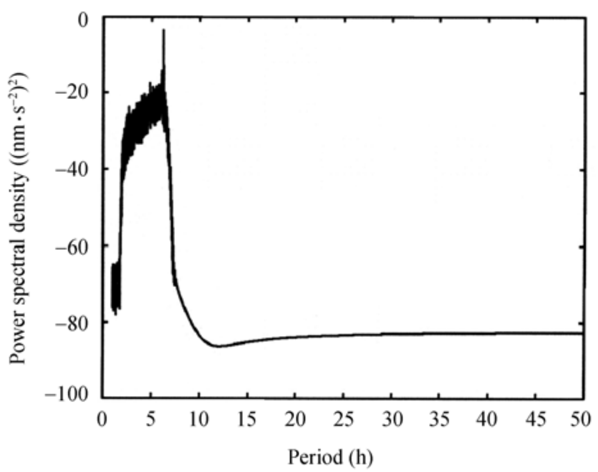

Fig. 6. Spectral results of the high frequency series when using multi-taper spectral estimation technique.

4 Parameters estimation of the determined signals

In order to estimate the oscillation parameters (i.e. the signal periods and magnitudes) in gravity residual series determined when using the WTFS technique (Fig. 5), the least squares adjustment of the Householder Transform has been applied. The corresponding calculation formula is given as

$$SL_t = a + \sum_{k=1}^{3} b_k \sin(2\pi t / P_k + \varphi_k) + \varepsilon_t, \qquad (4)$$

where P_k, b_k, and φ_k are the period, amplitude, and phase lag of gravity oscillations respectively, and a is a constant. Considering the gravity signals detected in sub-tidal band are unstable, the best mean yearly periods and amplitudes of these signals are identified and estimated by using a method of trial and errors. Based on analysis results of the WTFS, the initial period values are taken as 6.0, 5.0 and 4.0 h respectively.

The estimated periods and amplitudes of several special signals in sub-tidal wave band of gravity residual series calculated from least square estimation of eq. (4) are given in Table 1. The RMS column represents the root mean square values calculated from least square residuals to the high pass series in sub-tidal band. For comparing convenience, the RMS values calculated from the time series before least square fit are also given in brackets of Table 1. It is found from Table 1 that average yearly periods (*P*) and amplitudes (*Amp*) in different time spans are stable, and the estimated yearly mean amplitudes are in nGal level, at the same level as that of instrumental observation precision. The analysis shows that the change in *RMS* values is small before and after the data fitting.

However, from the analysis shown in Fig. 5, it is found that the periods and amplitudes of the special signal in sub-tidal band have indeed the feature of random walks in time process, showing that there are no stable provoking sources of gravity signals near the spectral peaks if these signals correspond to the possible Slichter modes. Except for the oscillation signals in periods of 4, 5 and 6 h, no other signals are found from our WTFS results. This is a little bit different from those in previous publications[13,15]. Furthermore, the amplitudes of the instantaneous signals are relatively obvious at tenth nGal level, and the frequency characteristic is unstable. Therefore, if there exist the signals relating to the translational oscillations of the ESIC, the provocation mechanics are probably not steady.

5 Discussion and conclusions

In the present study, differing from the conventional DFT method, the WTFS technique is applied successfully for the first time in a five-year length of SG gravity observations at Wuhan Station in order to detect the weak gravity signals associated possibly with the translational oscillations of the ESIC.

The results demonstrate that in sub-tidal band, the gravity signal features changed as time and frequency are revealed. For the remaining signals of the monthly, fortnight, diurnal and semi-diurnal waves in SG gravity residuals, the determined periods and amplitude are given as (27.55, 13.65, 1.0, 0.5d) and (0.161±0.008, 0.130±0.008, 0.123±0.008 and 0.036±0.008 μGal) respectively. These signals may relate to various geophysical reasons such as background noise, change in groundwater, shallow sea tides, regional pressure and imperfect monthly and fortnight synthetic tidal gravity models.

The MSF technique with high resolution is used effectively to eliminate low frequency signals in order to avoid the influence of the strong spectral power energy at periods from semi-diurnal to monthly down to the detected signals in sub-tidal wave band. The WTFS is successfully applied in this study, and the results indicate that the weak gravity signals in period of about 4,

Table 1 Estimated periods and amplitudes of several special signals in sub-tidal band

Year	*P* (h)	*Amp* (μGal)	*P* (h)	*Amp* (μGal)	*P* (h)	*Amp* (μGal)	*RMS* (μGal)
1998	6.0	0.0033±0.0003	4.8	0.0023±0.0003	4.0	0.0014±0.0003	0.0258 (0.0260)
1999	6.0	0.0028±0.0003	5.2	0.0018±0.0003	4.1	0.0016±0.0003	0.0269 (0.0270)
2000	6.0	0.0039±0.0003	5.2	0.0019±0.0003	4.0	0.0011±0.0003	0.0245 (0.0247)
2001	6.2	0.0032±0.0003	5.0	0.0021±0.0003	4.2	0.0014±0.0003	0.0264 (0.0266)
2002	6.0	0.0029±0.0004	5.2	0.0030±0.0004	3.5	0.0017±0.0003	0.0387 (0.0388)

5 and 6 h at nGal level are found, and their amplitudes are at the same level as instrumental precision. The further analysis shows that the frequency characteristics of the signals are non-stationary with a random walk property. It indicates that no stable provoking source of the signals relates to the possible Slichter modes, thus the translational oscillations of the ESIC do not probably and steadily exist and are only a very low amplitude.

It should be indicated here that this work is just a very beginning, the obtained conclusion is also preliminary. In practice, there exist obvious many difficulties for us to search accurately for the translational oscillations of the ESIC, as (1) there is no well-accepted theoretical model nowadays due to the lack of knowledge in Earth's deep interior; (2) although the SG ideal precision in instrumental laboratory is at 10^{-11} m • s^{-2} level, the gravity signals induced by the translational oscillations of the ESIC to be determined are very weak, and also, there exist surely the station background noises, especially, the determined SG signals at sub-tidal band may be often seriously contaminated by station background noises and some other factors and (3) the mechanical, sources of the translational oscillations of the ESIC are not yet clear for nowadays, for instance are they induced by large earthquakes in Earth's deep interior or by strong electronic-magnetic spiral field induced by iron components and high temperature in liquid outer core and together with topography coupling force toques at the inner core and outer core boundaries due to the Earth's rotation? Therefore the deep study and reliable conclusions will depend on the confidential theoretical model, the accumulating global long period high precision SG observations and the advanced data preprocessing techniques.

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