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PRELIMINARY APPLICATION OF SUPERCONDUCTIVE GRAVITY TECHNIQUE ON THE INVESTIGATION OF VISCOSITY AT CORE-MANTLE BOUNDARY

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Abstract The retrograde rotational eigenmodes are produced by the interaction between solid mantle and liquid core in a rotating elliptical Earth, usually called free core nutation (FCN). The FCN quality factor (Q value), which depends on viscosity at the core-mantle boundary (CMB), can effectively reflect the characteristics of energy dissipation at the CMB. The viscosity of the CMB is estimated for the first time based on the FCN quality factor Q values determined from stacking 27 high-rate sampling and high-precision tidal gravity observations at 23 stations in the Global Geodynamics Project (GGP) network around the world. The numerical results show that the dynamic viscosity can reach the order of 10^3 Pa·s. It is in good agreement with the most recent values deduced from VLBI observations by Smylie and Palmer. This result indicates that gravity is one of efficient techniques for the investigation of the Earth deep internal structure.

Key words Superconducting gravimeter, Free core nutation, Quality factor, Viscosity at the CMB

1 INTRODUCTION

The viscosity near the core-mantle boundary (CMB) is an important parameter for the determination of the geodynamic processes in the Earth's interior and for the study of the origin of the Earth's magnetic field. As a matter of fact, the knowledge of the properties in the Earth's interior (such as the elasticity, density, viscosity, pressure and gravity, etc.) is mainly obtained by the seismic inversion technique[1]. Currently it is difficult to model accurately the viscosity in the Earth's deep interior (especially near the CMB) by means of high temperature and high pressure experiments under laboratory conditions. As early as 1926, it was found that a P-wave velocity anomaly exists at the bottom of the mantle, which is relates to the viscosity at the CMB[2]. As the CMB is the coupling region between the solid mantle and liquid core, it is critical to explore its structure and its coupling mechanism for understanding the dynamical characteristics of the Earth's liquid core[3~5]. Therefore, the accurate estimation of the viscosity near the CMB is significant to understanding the properties of this medium[6~8].

Due to the interaction between the solid mantle and the liquid core for a rotating elliptical Earth, a retrograde rotational eigenmode, known as free core nutation (FCN), will occur in an inertial space reference frame. It appears as nearly diurnal free wobble (NDFW) in a terrestrial reference frame[7,9,10]. The existence of the FCN will produce an obvious resonance enhancement in the diurnal tidal spectrum (such for the waves, such as P_1, K_1, ψ_1 and φ_1, with frequencies close to its eigenfrequency). Based on this phenomenon, the FCN resonance parameters, including the eigenfrequency, quality factor, resonance strength and so on, can then be retrieved from ground-based high-precision tidal gravity observations. The FCN quality factor (Q value) reflects various effects, such as Earth's anelasticity, tidal friction, dissipative and topographic coupling mechanisms at the CMB. Among them the influence of the dissipative coupling is usually considered as the

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main part. The dissipative coupling includes two parts, the viscous coupling and the electromagnetic coupling. Smylie and Palmer[6,11] demonstrate that the influence of viscous coupling is a few orders larger than the effect of electromagnetic coupling. Therefore only the influence of viscous coupling will be considered in this study.

It is found that the motion and properties of the liquid outer core play an extremely important role in the evolution of the Earth, especially it relates to the formation and evolution of the magnetic field of the Earth, and they are significant in understanding the dynamic process of the Earth's interior. The dynamical characteristics of the Earth's liquid outer core mainly depend on some geophysical parameters, such as the diffusion coefficient, viscosity, thermal conductivity, electrical conductivity and so on[2,4]. The viscosity of the outer core can be estimated using various techniques, including theoretical simulations, high -temperature and high-pressure experiments in laboratory and measurements based on the seismology, geomagnetism and geodesy techniques. However, there exists obvious discrepancy in their numerical results with a wide range from 10^{-4} to 10^{11} Pa·s. There is now no generally accepted conclusion due to the relatively large uncertainty of each technique.

The boundary layer theory of the Earth's interior was brought into the investigation domain of energy dissipation near the CMB by Palmer and Smylie[11] and Smylie and Palmer[6]. They deduced a relationship between Q value of the FCN and viscosity at the CMB. At the same time the FCN phenomenon was studied and the viscosity at the CMB was further estimated from the continuous VLBI nutation observations for more than 23 years delivered by the GSFC (Goddard Space Flight Center) and the USNO (United States Naval Observatory). A spectral analysis technique was applied to consecutive data sets with a common length of 2000 days and 80% overlapping. Important parameters such as resonant strength and quality factor Q values are obtained and the attenuation of the FCN was determined. Their results provide us with an important theoretical basis for the study of the viscosity at the CMB from tidal gravity observations.

The development of the superconducting gravimeter (SG) and the availability of long periods of continuous tidal observations at many stations around the world with this instrument allow the study in this domain possible. The SG is a kind of relative gravimeter, developed by the American GWR Company based on superconductivity principle. It possess many outstanding advantages, such as an extremely wide measuring range, extremely low noise level, very low drift and high stability and sensitivity. It can be effectively used to detect weak gravity signals caused by the dynamical phenomena from the Earth's interior, such as Earth's free oscillations, the FCN of the liquid core, the translational oscillations of the solid inner core, and the Earth's rotation[12,13]. The Global Geodynamics Project (GGP)[14] was organized by the group of the Study of the Earth's Deep Interior (SEDI) under the IUGG in 1997. The investigation of the FCN is one of the important targets for SG gravity observations[14~16].

In this study, 27 data series of high-precision tidal gravity observations recorded with 23 superconducting gravimeters in the GGP network are used. Based on the international standard data processing method, the Earth tidal parameters which reflect properties of the Earth are estimated as a first step. According to various factors such as the standard deviation of the estimated tidal parameters and the regional distribution of the stations, several stacking combinations of the series are selected in order to obtain accurate FCN parameters. The stacking technique on globally distributed observations in different regions is an efficient way to eliminate local environmental perturbations around stations, and provide thus more accurate results. Furthermore, the viscosity near the CMB is estimated based on the FCN quality factor with reference to the theory of Smylie and Palmer[6].

2 SG OBSERVATIONS

The Global Geodynamics project (GGP) began in 1997, which suggested that every partner should use the same GWR superconducting sensor, an uniform data acquisition system and low pass filter, and same software packages for data processing[15]. For this research, the 27 high-precision and high-density data with 1-minute-sampling rate recorded at 23 stations equipped with superconducting gravimeters were obtained by international data exchange inside the GGP network (Table 1). During the data processing, some disturbances such as jumps, gaps, spikes, steps and earthquakes are removed by using the T-soft recommended by the International Center of Earth's Tides[16]. Then the numerical filtering process is performed to the minute gravity and auxiliary pressure data, sampling these data again to collect 1-hour data. In order to provide an assessment and comparison of data quality for different stations, the standard deviations of the tidal analysis results are also investigated. The lower standard deviation means a small station background noise with high data quality. The basic information, e.g., station names, data sections, observing periods, atmospheric gravity admittance (Adt) and standard deviations (Stv) in the tidal bands are listed in Table 1.

Table 1 Basic information of global SG observations used in this paper

No.	Station name	Data period y-m-d∼y-m-d	Data length (in day)	Adt (10^{-9}m·s^{-2}/hPa)	Stv (10^{-9}m·s^{-2})
1	Bad Homburg/Germany/lo	2001-02-12∼2007-04-04	2222.417	–3.36762±0.00439	0.783
2	Bad Homburg/Germany/up	2002-12-05∼2007-10-31	2218.250	–3.33209±0.00468	0.835
3	Bandung/Indonesia	1997-12-19∼2003-06-30	1104.250	–4.58912±0.05786	2.938
4	Brussels/Belgium	1982-04-21∼2000-09-22	6691.542	–3.46422±0.00453	1.641
5	Boulder/America	1995-04-01∼2003-04-30	788.000	–3.51192±0.00640	1.109
6	Cantley/Canada	1989-11-07∼1995-12-31	1565.250	–3.22972±0.00639	1.352
7	Canberra/Australia	1997-07-01∼2007-04-18	3429.333	–3.35420±0.00650	1.019
8	Concepcion/Chile	2002-12-05∼2007-10-31	1584.833	–3.76821±0.00957	1.156
9	Esashi/Japan	1997-07-01∼2004-02-25	2274.125	–3.65408±0.00736	1.491
10	Kamioka/Japan	2004-10-22∼2007-05-31	901.375	–2.89824±0.01176	1.310
11	Kyoto/Japan	1997-07-01∼2002-07-31	1532.750	–3.06935±0.02809	3.691
12	Matsushiro/Japan	1997-05-01∼2006-06-30	3226.458	–3.51907±0.00458	1.009
13	Medecina/Italy	1998-01-01∼2007-05-31	3429.917	–3.52721±0.00470	0.792
14	Membach/Belgium	1995-08-04∼2007-10-30	4407.833	–3.30243±0.00327	0.878
15	Metsahovi/Finland	1997-07-01∼2007-10-31	3542.542	–3.68779±0.00563	1.485
16	Moxa/Germany/lo	2000-01-01∼2007-12-31	2820.792	–3.28889±0.00363	0.698
17	Moxa/Germany/up	2000-01-01∼2007-12-31	2797.125	–3.33087±0.00331	0.633
18	NY-Alesund/Norway	1999-09-20∼2007-04-30	2412.833	–4.57357±0.01425	2.954
19	Potsdam/Germany	1992-06-30∼1998-10-08	2250.083	–3.31298±0.00420	0.856
20	Strasbourg/France	1997-03-01∼2007-12-31	3764.125	–3.40345±0.00302	0.687
21	Sutherland/S.Africa/lo	2000-03-27∼2006-12-31	2470.333	–2.85139±0.01115	1.058
22	Sutherland/S.Africa/up	2000-10-23∼2006-12-31	2260.667	–2.15437±0.01122	1.024
23	Syowa/South Pole	1997-07-01∼2000-12-31	1279.333	–4.18430±0.00752	1.387
24	Vienna/Austria	1997-07-01∼2006-12-31	3402.375	–3.54410±0.00279	0.570
25	Wettzell/Germany/lo	1998-11-04∼2007-04-17	3000.417	–3.36616±0.00381	0.695
26	Wettzell/Germany/up	1998-11-04∼2007-04-17	2978.792	–3.45304±0.00405	0.735
27	Wuhan/China	1997-12-20∼2007-06-30	2599.000	–3.16160±0.00852	0.975

Note: The SGs in Bad Homburg, Moxa, Wettzell in Germany and Sutherland in S. Africa are double-sphere, lo represents lower sphere, up for upper sphere, Adt is the atmospheric gravity admittance, Stv is the standard deviation.

On the basis of data preprocessing, the response functions of gravity to the change in air pressure inside the tidal bands, i.e., the atmospheric gravity admittance, are evaluated by regression analysis technique. From Table 1, it is found that the atmospheric gravity admittance is variable from station to station. The minimum value (absolute) is -2.90×10^{-9} m·s^{-2}/hPa, if we exclude Sutherland and the maximum value (absolute) is -3.77×10^{-9} m·s^{-2}/hPa if we exclude the stations Bandung in Indonesia and Syowa in the south pole. Sun et al.[17] obtained an atmospheric gravity admittance as of -3.603×10^{-9} m·s^{-2}/hPa through the theoretical simulation based on standard atmospheric law and atmospheric cylinder distribution model. The comparative analysis indicates that for most stations, the measured atmospheric gravity admittances are close to the value obtained from the theoretical model. However, as mentioned above, there exist large discrepancies between measured and theoretical values at several stations. They might be caused by ground temperature or other disturbing signals mixed up with pressure observations. After computation of the atmospheric gravity admittance, the pressure influence mixed in tidal gravity observations can be eliminated when determining tidal gravity parameters. The tidal gravity parameters, amplitude factors and phase differences, can be accurately determined. The study shows that it is very important to eliminate effectively atmospheric effects to enhance the accuracy of the estimated of FCN parameters[7,10].

The oceanic tidal signal is the most important perturbation in tidal gravity observations. The influence of ocean tides on gravity can be divided into three parts, i.e., the direct Newtonian attraction of the tidal mass, the elastic deformation of the Earth under the mass loads and the additional effects arising from redistribution of the mass in the Earth's interior due to deformation. Therefore, the loading effects are also dominant error source in estimation of the high accuracy FCN parameters[18]. Since the oceanic tides have the same driving mechanism and similar spectral pattern as body tides, it is impossible to separate them by harmonic analysis. Therefore the oceanic influence should be well simulated using an ocean tidal model. In this study, the recent global oceanic models Fes04 and convolution integral technique are used to calculate the loading vectors of 8 major tidal waves. The Fes04 ocean model are now the one of most precise models. They are constructed by means of the finite element numerical technique with combination of tide gauge observations at coastal, island and sea bottom points, and they include also the data of orbit crossing points of T/P satellite in deep-sea area.

The loading correction vectors computed to tidal gravity for the major diurnal waves are given in Table 2.

Table 2 Loading correction vectors for some major tidal waves in diurnal band estimated from ocean tidal model Fes04

		Q_1		O_1		P_1		K_1	
No.	Station	Amplitude (10^{-8} m·s^{-2})	Phase (°)	Amplitude (10^{-8} m·s^{-2})	Phase (°)	Amplitude (10^{-8} m·s^{-2})	Phase (°)	Amplitude (10^{-8} m·s^{-2})	Phase (°)
1	Bad Homburg/Germany	0.0594	–172.27	0.1490	160.06	0.0555	53.24	0.1758	54.67
2	Bandung/Indonesia	0.3092	–58.77	1.4988	–80.39	0.7078	–95.89	2.0697	–95.01
3	Brussels/Belgium	0.0582	–170.91	0.1302	152.65	0.0694	58.23	0.2263	59.90
4	Boulder/America	0.1283	76.81	0.8312	69.79	0.4283	54.29	1.2829	54.09
5	Cantley/Canada	0.0552	68.65	0.4365	51.32	0.1987	46.76	0.5844	47.09
6	Canberra/Australia	0.1932	140.73	0.7003	139.88	0.2638	83.91	0.7917	83.05
7	Concepcion/Chile	0.3372	–118.40	1.5755	–133.70	0.6809	–158.42	2.0648	–158.61
8	Esashi/Japan	0.4291	32.00	2.0134	25.00	0.8310	7.29	2.5196	7.18
9	Kamioka/Japan	0.3185	23.32	1.4595	15.61	0.6171	–1.26	1.8854	–1.37
10	Kyoto/Japan	0.3462	19.80	1.6050	11.21	0.6731	–5.67	2.0672	–5.74
11	Matsushiro/Japan	0.3409	24.93	1.5732	17.42	0.6624	0.13	2.0201	0.03
12	Medecina/Italy	0.0502	168.27	0.1644	151.36	0.0706	79.64	0.2122	81.15
13	Membach/Belgium	0.0602	–169.74	0.1440	158.42	0.0646	55.27	0.2069	56.69
14	Metsahovi/Finland	0.0847	153.48	0.1741	132.21	0.0342	–37.88	0.0877	–35.40
15	Moxa/Germany	0.0600	–178.49	0.1520	157.37	0.0452	50.89	0.1439	52.72
16	NY-Alesund/Norway	0.1560	154.01	0.2546	138.68	0.1622	–87.76	0.4715	–89.37
17	Potsdam/Germany	0.0643	177.67	0.1586	155.38	0.0432	42.79	0.1388	45.54
18	Strasbourg/France	0.0595	–167.20	0.1583	164.96	0.0598	59.80	0.1885	60.52
19	Sutherland/S.Africa	0.0976	–138.37	0.3542	–163.34	0.0802	71.48	0.2531	72.07
20	Syowa/South Pole	0.5224	–163.66	2.1816	–168.74	0.5286	–176.45	1.5860	–176.63
21	Vienna/Austria	0.0543	172.76	0.1455	157.48	0.0294	75.65	0.0955	77.08
22	Wettzell/Germany	0.0567	179.91	0.1493	158.67	0.0402	59.84	0.1281	61.40
23	Wuhan/China	0.1419	–1.56	0.6085	–20.20	0.1867	–35.06	0.5829	–31.96

The numerical results indicate that at a given location the influences of oceanic tides on tidal gravity waves O_1 and K_1 are generally greater than those on tidal waves P_1 and Q_1, which are as large as 10^{-8} m·s^{-2}. For the coastal stations, such as Conception in Chile and Syowa in the south pole, the loading magnitude for waves O_1 and K_1 reaches a level of 2.0×10^{-8} m·s^{-2}. On the other hand, at most of the Japanese stations the loading magnitude K_1 is on a similar level. In addition, the loading effects of the oceanic tides at inland stations even far away from the sea cannot be ignored either. Taking Wuhan as an example, which is located at about 600 km from the coastline, the amplitude and phase of the loading vectors for O_1 and K_1 are up to (0.61×10^{-8} m·s^{-2}, –20.2°) and (0.58×10^{-8} m·s^{-2}, –32.0°), respectively. The European stations have a very low loading magnitude. The previous investigations show that it is very important to eliminate effectively the loading effects in order to improve the accuracy of the estimating FCN parameters[18].

Considering that, in diurnal band, the oceanic models provide us only with co-tidal maps of 4 waves $(Q_1, O_1, P_1$ and $K_1)$, and that no ocean tidal information for waves ψ_1 and φ_1 with frequencies close to the FCN is available, the loading correction for tidal waves ψ_1 and φ_1 are interpolated with the linear regression method[18] to eliminate the loading effects for these small waves. The used formulas are[7]

$$\begin{cases} L(\sigma)\cos(\lambda(\sigma))/(Th(\sigma)R(\sigma)) = a_1 + a_2\sigma + a_2\sigma^2 \\ L(\sigma)\sin(\lambda(\sigma))/(Th(\sigma)R(\sigma)) = b_1 + b_2\sigma + b_2\sigma^2, \end{cases} \tag{1}$$

where $Th(\sigma), L(\sigma)$ and $\lambda(\sigma)$ represent the height of equilibrium tide with frequency σ, amplitude and phase of gravity variations caused by the oceanic tide loading, respectively; $R(\sigma)$ is a parameter which describes the effect of the FCN resonance on oceanic tide with frequency σ, and a_1, a_2, a_3, b_1, b_2 and b_3 are the regression parameters in order to determine.

3 FIT OF FCN PARAMETERS

The observation equations are built from the hourly tidal gravity series at each station and the tidal parameters (amplitude factors and phase differences) of the wave groups are estimated together with their standard deviation using the Eterna software, a standard tidal analysis software[19]. A high-precision tide-generating potential expansion with 1200 partial waves in order to Tamura is used[20]. Thirteen tidal components $(\sigma_1, Q_1, \rho_1, O_1, NO_1, \pi_1, P_1, K_1, \psi_1, \varphi_1, \theta_1, J_1, OO_1)$ are separated in the diurnal band for each station. The numerical results show that the accuracy of the amplitudes reaches 0.1×10^{-9} m·s^{-2}. The following section illustrates that it is extremely important for the detection of the FCN resonance phenomena to use high-precision tidal parameters.

It is well known that the FCN phenomena can lead to an obvious resonance enhancement in tidal diurnal observations (such as $P_1, K_1, \psi_1, \varphi_1$ and so on). Therefore, the FCN resonance parameters can be retrieved by means of the ground-based high-precision tidal gravity observations including the complex frequency σ, complex amplitude factors, quality factor and so on. The change in theoretical tidal amplitude factor in the diurnal band as a function of frequency σ can be described by the following formula[7,10]

$$\delta_{\mathrm{th}}(\sigma) = \delta_0 + \frac{\tilde{a}}{\sigma - \tilde{\sigma}_{\mathrm{nd}}}, \tag{2}$$

where δ_0 is the amplitude factor independent on frequency, which is not influenced by the FCN resonance, $\tilde{a} = a^{\mathrm{R}} + \mathrm{i}a^{\mathrm{I}}$ is the complex resonance strength related to the geometric shape of the Earth and the rheological properties of the Earth's mantle, and $\tilde{\sigma}_{\mathrm{nd}} = \sigma^{\mathrm{R}}_{\mathrm{nd}} + \mathrm{i}\sigma^{\mathrm{I}}_{\mathrm{nd}}$ denotes the complex eigenfrequency of the FCN, with R and I representing its real and imaginary part, respectively. Then the quality factor Q value and the FCN eigenperiod are expressed as $Q = 0.5\sigma^{\mathrm{R}}_{\mathrm{nd}}/\sigma^{\mathrm{I}}_{\mathrm{nd}}$ and $T_{\mathrm{FCN}} = \mathit{\Omega}/(\sigma^{\mathrm{R}}_{\mathrm{nd}} + \mathit{\Omega})$, here $\mathit{\Omega}$ is the sidereal frequency of the Earth's rotation.

With respect to other waves, the wave O_1 has a higher accuracy as it has a relatively large amplitude. On the other hand, it is less influenced by the resonance (only in the order of magnitude 10^{-4}) since its frequency is far away from the FCN frequency. Therefore, the wave O_1 can be used as the reference value for the computation[9]. In order to reduce the effects of systematic discrepancies of the tidal observation, e.g. calibration errors, and local environmental disturbance on the fitted resonance parameters of the FCN, we subtract the wave O_1 signals from both sides of Eq.(2). Then the fitting model of the FCN parameters is given as

$$\tilde{\delta}(\sigma) - \tilde{\delta}(\sigma_{O_1}) = \frac{\tilde{a}}{\sigma - \tilde{\sigma}_{\mathrm{nd}}} - \frac{\tilde{a}}{\sigma_{O_1} - \tilde{\sigma}_{\mathrm{nd}}}. \tag{3}$$

To get the optimal values of $\tilde{a}$ and $\tilde{\sigma}_{\mathrm{nd}}$, we minimize the function f described as follows:

$$f = \sum_{\sigma} p(\sigma) \left| [\tilde{\delta}(\sigma) - \tilde{\delta}(\sigma_{O_1})] - \left[\frac{\tilde{a}}{\sigma - \tilde{\sigma}_{\mathrm{nd}}} - \frac{\tilde{a}}{\sigma_{O_1} - \tilde{\sigma}_{\mathrm{nd}}} \right] \right|^2, \tag{4}$$

where $p(\sigma) = 1/(\varepsilon \times |\sigma - \sigma_{\mathrm{nd}}^{\mathrm{R}}|)$ is the weight function of the tidal parameter with frequency σ, with ε the standard deviation of the gravity amplitude factor. Here $\sigma_{\mathrm{nd}}^{\mathrm{R}}$ is an unknown parameter. Previous studies indicate that we can consider the FCN period around 430 sidereal days. The corresponding FCN eigenperiod is –15.076°/h, which can be used as a first approximation in choosing the weights $p(\sigma)$.

4 RELATIONSHIP BETWEEN QUALITY FACTOR (Q) AND VISCOSITY AT THE CMB

The quality factor Q_{w} of the FCN is usually defined as 2π times of the ratio in the total energy T to the energy E dissipated per cycle at a boundary layer. The angular frequency of the FCN is close to the negative angular velocity of the Earth's rotation[21,22]. According to the theory of Smylie and Palmer[6], the velocity field under the FCN in a spherical coordinates (r, θ, ϕ) system can be expressed as

$$v = -\hat{\theta} A r \sin(\phi + \mathit{\Omega} t) - \hat{\phi} A r \cos\theta \cos(\phi + \mathit{\Omega} t), \tag{5}$$

where A is the amplitude of the FCN, r, θ and ϕ represent the radius, colatitude and longitude of the spherical Earth, respectively. Assuming that the boundary layer is a small fraction of the radius in thickness, then the movement equations of the velocity components (v_θ, v_ϕ) for a point mass at boundary layer can be written as

$$\frac{\partial v_\theta}{\partial t} - 2\mathit{\Omega} v_\phi \cos\theta = \nu \frac{\partial^2 v_\theta}{\partial r^2}, \qquad \frac{\partial v_\phi}{\partial t} + 2\mathit{\Omega} v_\theta \cos\theta = \nu \frac{\partial^2 v_\phi}{\partial r^2}, \tag{6}$$

where ν is the kinematic viscosity. The velocity components (v_θ, v_ϕ) can then be obtained by solving Eq.(6). According to Smylie and Palmer[6], the principle compressive stresses at the boundary layer are

$$\sigma_{r\theta} = \eta \frac{\partial \mathrm{Re}(v_\theta)}{\partial r}, \qquad \sigma_{r\phi} = \eta \frac{\partial \mathrm{Re}(v_\phi)}{\partial r}, \tag{7}$$

where η is the dynamic viscosity ($\eta = \rho_0 \nu, \rho_0$ is the density of the boundary layer), $\mathrm{Re}(v_\theta)$ and $\mathrm{Re}(v_\phi)$ are the real parts of velocity components (v_θ, v_ϕ), respectively.

Besides, the rate of dissipation of the energy per unit area is a function of the velocity of the mass point at boundary layer, which is of the form

$$\frac{\mathrm{d}e}{\mathrm{d}r} = v_\theta \sigma_{r\theta} + v_\phi \sigma_{r\phi}. \tag{8}$$

If the amplitude of the FCN of the outer core with respect to the inner core is A_a and that of outer core with respect to the mantle is A_b, integrating over the whole boundary layer, the total energy dissipation per cycle in both boundaries is

$$E = \frac{2\pi}{\mathit{\Omega}} \frac{\mathrm{d}E}{\mathrm{d}t} = \frac{2}{35} \pi^2 (\rho_0(a) A_a^2 a^4 \sqrt{\nu_a} + \rho_0(b) A_b^2 b^4 \sqrt{\nu_b}) \rho \sqrt{\frac{2}{\mathit{\Omega}}} (9\sqrt{3} + 19), \tag{9}$$

where $\rho_0(a)$ and ν_a are the medium density and kinematic viscosity at the ICB ($r_0 = a$), respectively; $\rho_0(b)$ and ν_b are the medium density and kinematic viscosity at the CMB ($r_0 = b$), respectively. According to Palmer and

Smylie[11], the reciprocal of the overall quality factor of the FCN is the sum of the reciprocals of the quality factors at the top and bottom of the liquid outer core[11]:

$$\frac{1}{Q_{\mathrm{w}}}=\frac{1}{Q_a}+\frac{1}{Q_b}. \tag{10}$$

According to the definition of the quality factor and the expression of the energy dissipation ratio at the boundary layer, we get

$$\frac{1}{Q_a}=\frac{2\pi\rho_0(a)a^4\sqrt{2\nu_a/\mathit{\Omega}}(9\sqrt{3}+19)}{35I_{\mathrm{oc}}},\qquad \frac{1}{Q_b}=\frac{2\pi\rho_0(b)b^4\sqrt{2\nu_b/\mathit{\Omega}}(9\sqrt{3}+19)}{35I_{\mathrm{oc}}}, \tag{11}$$

where I_{oc} represents the inertial moment of the liquid outer core. Neglecting the perturbation of the Earth's solid shell rotation, and if the amplitude of the FCN of the outer core is B and that of the inner core is C, there are $A_b=B$ and $A_a=B-C$. Considering the total viscous coupling torque of the outer core with respect to the solid shell and inner core, and also ignoring the small effect induced from Chandler resonance[6,11], the relation between B and C can be deduced from the linear differential system constituted by the equation of motion of the outer core and that of the inner core as

$$C=\left(1+\frac{r_b}{r_a}\frac{I_{\mathrm{ic}}}{I_{\mathrm{c}}}\right)B,\qquad B-C=-\frac{r_b}{r_a}\frac{I_{\mathrm{ic}}}{I_{\mathrm{c}}}B, \tag{12}$$

where r_a and r_b are the coefficients of viscous coupling between the outer core and the inner core, the outer core and the shell, respectively, with $r_b/r_a=1/117.4$; I_{ic} and I_{c} represent the inertial moment of the inner core and that of total Earth's core. Substituting Eq.(12) into (9), yields the total energy dissipation in per cycle at the both boundaries:

$$E=\frac{2\pi}{\mathit{\Omega}}\frac{\mathrm{d}E}{\mathrm{d}t}=\frac{2}{35}\pi^2\rho_0(b)A_b^2b^4\sqrt{\frac{2\nu_b}{\mathit{\Omega}}}(9\sqrt{3}+19)\left[1+\frac{r_a}{r_b}\left(\frac{I_{\mathrm{ic}}}{I_{\mathrm{c}}}\right)^2\right]. \tag{13}$$

Therefore the total energy at the boundary layer is

$$T=\frac{1}{2}I_{\mathrm{oc}}A_b^2+\frac{1}{2}I_{\mathrm{ic}}\left[1+\frac{r_b}{r_a}\frac{I_{\mathrm{ic}}}{I_{\mathrm{c}}}\right]^2A_b^2=\frac{1}{2}I_{\mathrm{c}}\left[1+2\frac{r_b}{r_a}\left(\frac{I_{\mathrm{ic}}}{I_{\mathrm{c}}}\right)^2\right]A_b^2, \tag{14}$$

and the quality factor of the FCN is

$$Q_{\mathrm{w}}=2\pi T/E=\frac{35I_{\mathrm{c}}[1+r_b/r_a(I_{\mathrm{ic}}/I_{\mathrm{c}})^2]}{2\pi\rho_0(b)b^4\sqrt{2\nu_b/\mathit{\Omega}}(9\sqrt{3}+19)}. \tag{15}$$

Finally, the viscosity at the CMB can be retrieved from the above equation as

$$\nu=\frac{1225I_{\mathrm{c}}^2\mathit{\Omega}[1+2r_b/r_a(I_{\mathrm{ic}}/I_c)^2]}{8\pi^2\rho_0^2(b)b^8(9\sqrt{3}+19)^2Q_{\mathrm{w}}^2}, \tag{16}$$

Taking the radius of the outer core b=3480 km, the density at CMB $\rho_0(b)=10^4$ kg·m^{-3}, the Earth's rotational velocity $\mathit{\Omega}=7.292115\times10^{-5}$ s^{-1} and also taking the same values of the inertia moment for the inner core and the outer core as the one given by Smylie and Palmer[6], i.e., $I_{\mathrm{ic}}=6.16\times10^{34}$ kg·m^2, $I_{\mathrm{oc}}=911.79\times10^{34}$ kg·m^2 then the total moment of inertia of the core is $I_{\mathrm{c}}=I_{\mathrm{oc}}+I_{\mathrm{ic}}=917.95\times10^{34}$ kg·m^2.

5 NUMERICAL RESULTS AND DISCUSSION

In this study, 27 tidal gravity observation series recorded with 23 superconducting gravimeters in the GGP network are used. Different numerical results of the FCN parameters are calculated from different combinations of the observations. First, the tidal gravity parameters (amplitude factors and phase differences for the various wave groups) with pressure correction are determined via the Eterna software for tidal analysis. Then according to the principle of vector superposition, the loading effects of oceanic tides are removed from the tidal gravity parameters. Finally, the gravimetric parameters after correction of air pressure and oceanic tides loading are used to estimate the FCN resonance parameters. The stacking technique is used for the estimation the FCN parameters considering a relative large dispersion of the quality factor determined at a single station. The stacking technique is effective for simultaneous reduction the dispersion of global stations and can improve the accuracy of the estimated FCN quality factor. In this study, different combinations of the observation series, including 5, 8, 10, 12, 15, 18, 21, 24 and 27 data sets (see Table 3 for details), are chosen to retrieve the FCN eigenperiods and quality factors. Each group corresponds to a given limit of the standard deviations given in Table 1. For example, the above mentioned combination of 5 series consists of the observations with the least standard deviation lower than 0.7×10^{-9} m·s^{-2}. For convenience of comparison, we stack also the observations located in the southern (6 series) and northern hemisphere (21 series), respectively. Considering the dense distribution in Europe, the observations of stations in Germany (7 series) and the whole Europe (14 series) are also stacked. The corresponding results are given in Table 3.

Table 3 FCN parameters and dynamic viscosities estimated using global SG observations

Data series	Stv.($\times10^{-9}$ m·s^{-2})	Eigen period (SD)	Quality factor Q	Viscosity (Pa·s)
5(No.16,17,20,24,25)	< 0.7	–428.6 (–423.2, –434.0)	12138 (7103, 41672)	2514
8(No.1,13,16,17,20,24–26)	< 0.8	–429.2 (–425.0, –433.5)	9489 (6624, 16720)	4114
10(No.1,2,13,16,17,19,20,24–26)	< 0.9	–429.4 (–426.0, –432.9)	9331 (6925, 14302)	4255
12(No.1,2,13,14,16,17,19,20,24–27)	< 1.0	–429.2 (–425.1, –433.4)	11331 (7512, 23040)	2885
15(No.1,2,7,12–14,16,17,19,20,22,24–27)	< 1.1	–432.1 (–426.6, –437.8)	14072 (7643, 88483)	1871
18(No.1,2,5,7,8,12–14,16,17,19–22,24–27)	< 1.2	–430.8 (–424.9, –436.9)	17626 (8268, ∞)	1192
21(No.1,2,5–8,10,12–14,16,17,19–27)	< 1.4	–431.3 (–425.2, –437.6)	23792 (9211, ∞)	654
24(No.1,2,4–10,12–17,19–27)	< 1.7	–432.1 (–426.1, –438.2)	33467 (10598, ∞)	331
27(No.1–27)		–431.9 (–424.8, –439.3)	32728 (9232, ∞)	346
Europe (14)		–430.1 (–424.7, –435.7)	11079 (6682, 32396)	3018
Germany (7)		–429.9 (–424.7, –435.3)	9313 (6067, 20020)	4271
Northern hemisphere (21)		–435.6 (–426.6, –444.9)	10966 (5346, ∞)	3081
Southern hemisphere (6)		–435.0 (–392.6, –487.7)	36079 (1912, ∞)	285
15 groups with positive Q		–431.2 (–426.6,–435.9)	10714 (6976, 23077)	3227

From the estimated quality factors (Q value) of the FCN, the viscosity at the CMB can be computed on the basis of expression (16). The corresponding results are also given in Table 3. It is found that the values of the dynamic viscosity near the CMB are in the range from 10^2 to 10^3 Pa·s by using corresponding Q estimations retrieved by stacking different combination of the SG observation series. As shown in the first row in Table 3, the viscosity estimated is 2514 Pa·s when stacking the groups of the 5 observations series at 4 stations (including Moxa in Germany, Strasbourg in France, Vienna Austria and Wettzell in Germany) with the least standard deviation (not exceeding 0.7×10^{-9} m·s^{-2}). By comparison, it is also found that the result is in relatively good agreement when stacking the groups up to 18 observation series with a standard deviation below 1.13×10^{-9} m·s^{-2}. While stacking more than 21 series and observations in southern hemisphere, the viscosity becomes one order of magnitude smaller. The analysis shows that this difference is caused by the poor data quality at some stations, for example Bandung in Indonesia, Kyoto in Japan and NY-Alesund in Norway are the three bad stations with standard deviations close to 3×10^{-9} m·s^{-2}. Bandung is located close to the equator and the amplitude of the diurnal waves becomes too small for a precise determination. Observations of Kyoto in Japan are strongly influenced by noise around the station. Ny Alesund is located a few ten meters from the sea so that the modeling of the ocean loading contribution is not accurate. When the observations with high background noise are used to estimate the FCN parameters, the Q value will be affected greatly.

On the other hand, by calculation of the FCN quality factors for each single station, we find 15 data sets among the 27 sets are positive. Most of them are located in Europe. In the southern hemisphere the unique station with a positive quality factor is at Canberra in central Australia. Most of the series from Asia are of negative quality factors. It is probably due to the large diurnal ocean loading observed in this part of the world (Table 2), which is much more difficult to correct than in Europe where this effect is very small. Stacking all the observation series with positive quality factors, yields a dynamic viscosity value of 3227 Pa·s.

It should be pointed out that the negative FCN quality factors are obtained from 12 observation series. It contradicts the basic physical meaning of the quality factor. Therefore, an estimation of the viscosity based on them has no physical meaning. The analysis shows that there are many reasons for negative quality factors[23]. The primary one is that the imaginary part of the FCN eigenfrequency used to calculate the quality factor is a small quantity, which is easily affected by environmental factors, such as uncertainty of global and regional ocean tidal models, influence of regional environment around a station, including background noise, variation of the air pressure and change of temperature and groundwater level. According to the geographical distribution of the stations with negative quality factors, the loading effect of ocean tides is likely to account for a large part of the problem.

Smylie and Palmer[6] presented values of the dynamic viscosity of 3038 and 2866 Pa·s calculated from the VLBI nutation observations provided by the GSFC and USNO, respectively (see Table 4). Similar values of the dynamic viscosity are obtained by stacking up to 18 SG observations with a standard deviation of the tidal parameters lower than 1.13×10^{-9} m·s^{-2} or 14 series in Europe, 7 in Germany and 21 in the northern hemisphere (see Table 3). From the stacking results of data sets grouped according to the standard deviation of the tidal harmonic analysis in Table 3, it is found that the results obtained by stacking 21, 24 and 27 series including observations with higher standard deviation disagree with those obtained using VLBI observations. This is mainly because these combinations include more series with big standard deviations. The scarcity of stations in the southern hemisphere and the poor location of many of them explain the discrepancy between the results of the both hemispheres. The area of best results is certainly Europe due to its very low diurnal tidal loading effects[16].

From the discussion above, the obvious conclusion is that similar viscosity values are obtained using ground-based high-precision tidal gravity registration and VLBI nutation observations. This conclusion is quite significant, as nowadays in laboratory conditions high-temperature and high-pressure experiments can not estimate accurately the viscosity in the Earth′s deep interior (especially at the CMB). A big dispersion can be also found when using other techniques compared to the results obtained under laboratory simulations. For the convenience of comparison, the dynamic viscosities in the range from 10^{-4} Pa·s to 10^{11} Pa·s obtained using different techniques are listed in Table 4.

From this table, it is found that there is a big discrepancy among the results obtained from VLBI observations, those obtained from high-temperature and high-pressure experiments and those obtained at an early stage by other techniques such as seismology and geomagnetism.

The viscosity values are in the range $10^2\sim10^3$ Pa·s deduced from VLBI observations, while larger than 10^6 Pa·s estimated by seismology and geomagnetism. Most of the viscosity results inferred from experiments at high temperature and high pressure are around 10^{-2} Pa·s and the minimum value is around 10^{-4} Pa·s. The only one exception comes from the extrapolated results based on the high temperature and high pressure experiment given by Brazhkin et al.[22] (2000). It provides us an approached value of the viscosity, but still one order lower, compared to the ground based tidal gravity and VLBI observations.

Certainly, there is still a big controversial on the study of the dynamic viscosity at the CMB[24], especially it is difficult to explain the large discrepancy between experiment in laboratory conditions and actual observations. Considering that results of high temperature and high pressure experiments are around $10^{-2} \sim 10^{-4}$ Pa·s, some scientists tried to explain this disagreement with eddy viscosity at CMB. In addition, the viscosities at top and bottom of the outer core as of 10^2 Pa·s and 10^{11} Pa·s are estimated using Arrhenius extrapolation to the result of laboratory experiments by Brazhkin[23], this viscosity is only one order lower than the value obtained by Smylie and Palmer[6] from VLBI observations and by this paper from SG observations. Therefore, it should be a significant contribution to the study of the viscosity at the CMB to give a reasonable explanation of this discrepancy.

Table 4 Dynamic viscosity at the core-mantle boundary

Subject	Author	Dynamic Viscosity (Pa·s)	Data, Method
Geodesy	Molodenskiy(1981)[1]	$\le 10^6$	Forced nutation of the Earth
	Gwinn et al.(1986)[1]	$< 5.4 \times 10^3$	Retrograde annual Earth nutation VLBI measurements
	Palmer & Smylie(2005)[11]	788(743)	VLBI measurements
	Smylie & Palmer(2007)[6]	3038(2866)	VLBI measurements
	This paper	2514	Gravity technology
Seismology	Sato & Espinosa(1967)[1]	8.6×10^{11}	Multiply reflected s-waves at CMB
Geomagnetism	Hide(1971)[1]	10^6	Magnetohydrodynamics
Theoretical physics	Poirier(1988)[1]	10^{-2}	Experiments under high temperature and high pressure
	Rutter(2002)[21]	10^{-4}	Experiments under high temperature and high pressure
	Brazhkin(2000)[22]	10^2(CMB)	Experiments under high temperature and high pressure
	Brazhkin(2000)[22]	10^{11}(ICB)	Experiments under high temperature and high pressure

6 CONCLUSIONS

In this study, we use 27 high-precision tidal gravity observations at 23 stations in GGP network. A unified data preprocessing method and international standard processing methods are utilized to estimate the tidal gravity parameters after correction of air pressure. Then these parameters are used to estimate the FCN parameters after correction of ocean tides loading. A stacking procedure is applied to selected groups of stations. On this basis, the viscosity at the CMB is estimated. The numerical results show that the dynamic viscosity estimated by high-precision tidal gravity observations reaches 10^3 Pa·s. It is of the same order as the latest values obtained from VLBI nutation observations. This result indicates that tidal gravity is one of the effective techniques for investigating the deep interior structure of the Earth. It is worth to point out that the FCN quality factor can be affected by many factors such as the measurement of phase differences of the gravimeter, models of the atmosphere and ocean tides loading, coupling models at the CMB and ICB and so on. Therefore, further studies should deserve to conduct based on high-precision FCN quality factors obtained by tidal gravity observations.

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